

can be implemented. This is a known result, which, translated to the notation of transform matrices, is

$$\tilde{P} = \tilde{A}_{1,D} \tilde{A}_{1,P} \tilde{A}_{2,D} \tilde{A}_{2,P} \dots \tilde{A}_{N,D} \quad (4)$$

where $\tilde{A}_{n,p}$ are permutation matrices to account for the delays (noting that we are using a noncausal representation). The matrices $\tilde{A}_{n,D}$ are block-diagonal (each block is $M \times M$) as

$$\tilde{A}_{n,D} = \text{diag} \{ \dots, R_n, R_n, R_n, \dots \} \quad (5)$$

with R_n having the same structure devised for each factor of the lossless transfer matrix [6]. Actually, R_n is an orthogonal matrix which can be implemented with $M - 1$ plane rotations. As an exception, R_1 is a general orthogonal matrix requiring $M(M - 1)/2$ plane rotations. Each rotation has an associated angle which is a free parameter in the design of a PUFB. This general decomposition into sparse matrices is not unique, but is minimal in terms of total number of rotations [6].

Orthogonality under time variations: Suppose the angles in each R_n in eqn. 5 are changed along the time index. More precisely, suppose it is written as

$$\tilde{A}_{n,D} = \text{diag} \{ \dots, R_n(k-1), R_n(k), R_n(k+1) \dots \} \quad (6)$$

Because only the angles are changed, each matrix is still orthogonal and PR is assured using the same flow graph for analysis or synthesis. The analysis-synthesis process is now described by a matrix \tilde{P} , where

$$\tilde{P} = \begin{pmatrix} P_0(k-1) & \dots & P_{N-1}(k-1) & 0 & 0 \\ 0 & P_0(k) & \dots & P_{N-1}(k) & 0 \\ 0 & 0 & P_0(k+1) & \dots & P_{N-1}(k+1) \end{pmatrix} \quad (7)$$

with

$$P(k) = [P_0(k) \ P_1(k) \ \dots \ P_{N-1}(k)] \quad (8)$$

This means that $P(k)$ contains the instantaneous filter bank impulse responses. In time-varying systems, we have to choose an index k and find the PR equations for it, noting that eqn. 1 is no longer valid. It is possible to show that the PR conditions for the time-varying case can be written as

$$\sum_{m=0}^{N-1-l} P_m(k) P_{m+l}^T(k-l) = \sum_{m=0}^{N-1-l} P_{m+l}(k) P_m^T(k+l) = \delta(l) I_M \quad (9)$$

for $l = 0, 1, \dots, N-1$, yielding $2N - 1$ independent matrix equations.

Conclusive analysis: With the proper choice of angles in the factorised form of $P(k)$, we are able to implement any steady PUFB, with M bands and filters with length up to L . Therefore, we can change the angles and switch between any two PUFBs. $P(k)$ has compact support (contains FIR filters) and will 'forget' past conditions, acting as the new filter bank, after some time. Clearly, there would be a transition region where none of the filter bank responses is achieved. At each instant we can calculate $P(k)$ which will provide the intermediary frequency response. However, the system is naturally PR, even in transitions, guaranteeing distortionless processing.

The main idea is to switch between two known PUFBs at a time. For this, each of them has to be expressed in its factorised form. To change the filter bank at instant k , we can set all angles in all $R_n(i)$ ($i \geq k$) to their new values. Finally, note that any orthogonal block transform ($L = M$) can be implemented, because a PUFB having filters whose length lies between M and L belongs to the set of all PUFBs whose filters have length L . This can be seen by constraining marginal elements of the filters to be zero. Therefore, it is possible to switch between lapped and block transforms among other possibilities. Even the identity matrix can be used, causing the

transform to be bypassed (copying input to output), while maintaining PR in transitions.

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R. L. de Queiroz and K. R. Rao (Electrical Engineering Department, University of Texas at Arlington, Box 19016, Arlington, TX, 76019, USA)

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COMMENT

PERFORMANCE OF CROSSBAR INTERCONNECTION NETWORKS IN THE PRESENCE OF 'HOT SPOTS'

R. Y. Awdeh and H. T. Mouftah

Pombortsis and Halatsis have considered the performance of crossbar interconnection networks in the presence of hot spot traffic conditions [A]. They have assumed an $N \times N$ crossbar operating in a synchronous mode, where each processor issues r requests to the shared memory per network cycle ($0 \leq r \leq 1$). A fraction h ($0 \leq h \leq 1$) of all references are aimed at a specific memory module MM_h (the hot spot memory module), i.e. each processor emits $r(1-h)$ requests uniformly over all N MMs, and rh requests to MM_h .

It was stated that the rate of requests at MM_h due to hot spot traffic is

$$P_h = 1 - (1 - rh)^N \quad (1)$$

and due to uniform background traffic is

$$P_u = 1 - \left[1 - \frac{r(1-h)}{N} \right]^N \quad (2)$$

Based on the assumption that requests are random and the requests generated by a processor are independent of the requests generated by another processor, it was concluded that P_h and P_u are statistically independent, and thus the total rate of requests at MM_h is

$$P_t = P_h + P_u - P_h P_u = 1 - (1 - rh)^N \left[1 - \frac{r(1-h)}{N} \right]^N \quad (3)$$

However, we think that the above procedure of obtaining P_t is incorrect, and that the statistical independence assumption between the rate of requests at MM_h due to hot spot traffic and those due to uniform background traffic is not valid.