ON FILTER BANKS WITH RATIONAL OVERSAMPLING†

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ABSTRACT

Despite the great popularity of critically-decimated filter banks, oversampled filter banks are useful in applications where data expansion is not a problem. We studied oversampled filter banks and showed that for some popular classes of filter banks it is not possible to obtain perfect reconstruction with rational (non-integer) oversampling ratios. Nevertheless, it is always possible to oversample the analysis filter bank by an integer factor, i.e. there will be a similar synthesis bank which would provide perfect reconstruction. The analysis is carried within a time-aliasing framework developed to analyze non-critically decimated filter banks.

1. INTRODUCTION

The theory of oversampled filter banks was extensively explored in [1]–[4]; however, uniform filter banks with non integer oversampling ratios were not clearly addressed in the literature. In this paper we use a time-domain approach to reveal more specific insights into the properties and constraints of uniform filter banks with a non integer oversampling ratio and perfect reconstruction. We consider particularly the case of filter banks used in lapped transforms [5].

Using the time-domain representation, we show that time alias is a limiting factor for the use of a non integer oversampling ratio; however, the use of redundant structures overcomes this problem. Fig. 1 shows an M-band filter bank decimated by a factor N. The term oversampled refers to filter banks where the number of bands is greater than the decimation factor, that is, $M > N$. For $M = N$, we have the critically sampled case. We also consider the use of two structures in parallel, each one critically sampled or oversampled. The term redundant is used to refer to the use of these structures in parallel.

The interest in oversampled filter banks is due to some improvements over critically decimated filter banks, such as, additional design flexibility, improved frequency selectivity, and improved noise immunity [3]. These improvements come, of course, at the expense of an increase in the computational cost caused by the need to process a larger number of subband signal samples per unit of time [6]. Thus, oversampled filter banks allowing an efficient implementation, such as oversampled lapped transforms, are of particular interest.

The case of rational oversampled filter banks has been more emphasized when using complex transforms like the discrete Fourier transform (DFT) or the short-time Fourier transform. One example, related to the short-time Fourier transform, is [7], which gave a parameterization of FIR paraunitary modulated filter banks with arbitrary rational oversampling ratios. However, with real-valued coefficient filter banks the possibilities for the oversampling ratios are not so clear. Examples are [2], [4] and [6]. All of them established conditions for perfect reconstruction using FIR filters. [4] and [6] explored the case of cosine modulated filter banks, and [6] looked also at linear phase filters, a property more pertinent to applications in image processing. Linear phase is not the case in [4] where the primary goal was application in audio and speech. Another example of a paper in the area of oversampled filter banks is [8], which mentioned that biorthogonal filter banks with perfect reconstruction and linear phase properties can be designed for an arbitrary decimation ratio.

In this paper we explore lapped transforms with non integer rational oversampling ratios, since for integer ratios we could refer to the works cited above.

Notation. Capital boldface letters are reserved for matrices, while lower case ones are reserved for vectors. In particular, $\mathbf{I}$ and $\mathbf{0}$ are the identity and null matrices. Their sizes may be indicated by subscripts when not clear from context. $\mathbf{J}$ is the reversing matrix, for example, $\mathbf{J}_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

2. TIME-DOMAIN REPRESENTATION

We consider an M-channel perfect reconstruction (PR) FIR filter bank whose filters are causal with maximum length equal to $L = KM$. In Fig. 1 we show a polyphase representation of an M-channel filter bank. The analysis filter bank is represented by the MIMO system $\mathbf{E}(z)$, which represents the set of M filters with impulse responses $\{h_i[\ell]\}$ for $0 \le i \le M-1$ and $0 \le \ell \le L-1$. A similar relation applies to the synthesis MIMO system $\mathbf{R}(z)$ and the synthesis filters $\{f_i[\ell]\}$. In an alternative matrix representation the filter coefficients can be grouped into lapped transform matrices $\mathbf{P}$ and $\mathbf{Q}$ of size $M \times L$, whose entries are

$$p_{ij} = h_i[L-1-j], \; q_{ij} = f_i[j]. \tag{1}$$

We can also divide $\mathbf{P}$ and $\mathbf{Q}$ into $M \times M$ matrices as

$$\mathbf{P} = [\mathbf{P}_0 \; \mathbf{P}_1 \; \cdots \; \mathbf{P}_{K-1}], \quad \mathbf{Q} = [\mathbf{Q}_0 \; \mathbf{Q}_1 \; \cdots \; \mathbf{Q}_{K-1}]. \tag{2}$$

The PR conditions [9], [10] establish that

$$\sum_{k=0}^{K-1-m} \mathbf{Q}_k^T \, \mathbf{P}_{k+m} = \sum_{k=0}^{K-1-m} \mathbf{Q}_{k+m}^T \, \mathbf{P}_k = \delta[m] \, \mathbf{I}_M. \tag{3}$$

In this time-domain representation, the polyphase diagram in Fig. 1 can be reformulated as in Fig. 2. Also, the L-tuple

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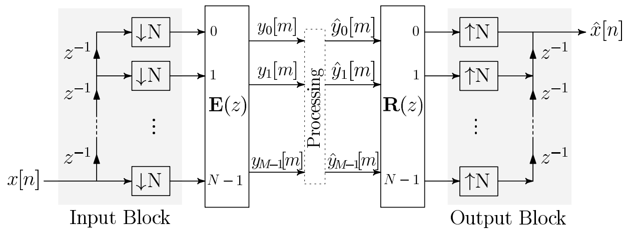

Figure 1. Polyphase representation of an M-band N-decimated filter bank.

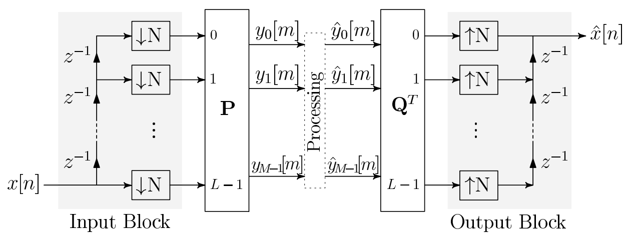

Figure 2. Block diagram of a general signal processing system using an M-band N-decimated lapped transform.

$\mathbf{x}_m$ can be defined as

$$\mathbf{x}_m = [\, x[m\,N] \quad x[m\,N+1] \quad \cdots \quad x[m\,N+L-1]\,]^T, \tag{4}$$

so that

$$\mathbf{y}_m = \mathbf{P}\,\mathbf{x}_m, \tag{5}$$

where $\mathbf{y}_m$ is the vector with the M subband samples at each sampling instant, that is,

$$\mathbf{y}_m = [\ y_0[m] \quad y_1[m] \quad \cdots \quad y_{M-1}[m]\ ]^T. \tag{6}$$

It is obvious that one cannot recover $\mathbf{x}_m$ from $\mathbf{y}_m$. If $\hat{\mathbf{x}}_m = \mathbf{Q}^T\mathbf{y}_m$, PR of $\mathbf{x}_m$ occurs because of the accumulation of all $\mathbf{x}_n$ which overlap with $\mathbf{x}_m$. When the decimation N is different from the number of bands M, the PR equation can be devised as

$$\sum_{n=kN,\ -L<n<L} \mathbf{Z}_n(\mathbf{Q}^T\mathbf{P})\mathbf{Z}_n^T = \mathbf{I}_L, \tag{7}$$

where k is an integer, and $\mathbf{Z}_n$ is an $L \times L$ matrix defined by

$$[\mathbf{Z}_n]_{i,j} = \delta[n-i+j], \tag{8}$$

for $i = 0, 1, \ldots, L-1$, and $j = 0, 1, \ldots, L-1$, where $[\mathbf{Z}_n]_{i,j}$ means the element at the ith row and jth column of the matrix $\mathbf{Z}_n$, and $\delta[n]$ is the unit impulse sequence [11]. For example, with $L=4$ and $n=1$,

$$\mathbf{Z}_1 = \mathbf{Z}_{-1}^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{9}$$

Note that the product with $\mathbf{Z}_n$ and $\mathbf{Z}_n^T$, in the left side of (7), shifts the matrix $\mathbf{Q}^T\,\mathbf{P}$ down along the main diagonal, for n positive, and up along the main diagonal, for n negative. One should also note that in (7) the conditions for PR can appear repeated.

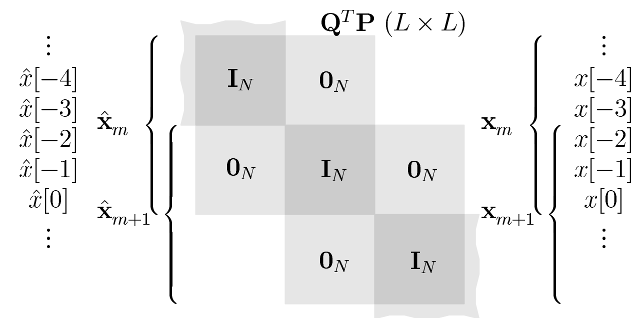

Figure 3. Illustration of a lapped transform processing with $L = 2\,N$.

We can use the example shown in Fig. 3 to better visualize the general idea behind (7). In this example $L = 2N$, so that the product $\mathbf{Q}^T\,\mathbf{P}$ is superposed with two of its blocks of order $N \times N$. To have PR this superposition must be equal to an identity matrix of order $L \times L$.

Now we will use the time domain notation introduced above to determine the conditions for perfect reconstruction in a filter bank with non integer oversampling ratio. We will focus our attention on a few lapped transforms.

3. LINEAR PHASE FILTER BANKS

In the case that the filters have linear phase, that is, the transform bases are either symmetric or anti-symmetric [12], [13], there are many more degrees of freedom than cosine modulated filter banks. However, it is still possible to explore properties of oversampled linear phase filter banks. For this, we will limit ourselves to analyze a short-length $L = 2M$ system of even bands, i.e. a system with the following factorization

$$\mathbf{P} = \begin{bmatrix} \mathbf{U} & \mathbf{0} \\ \mathbf{0} & \mathbf{V} \end{bmatrix} \begin{bmatrix} \mathbf{D}_e - \mathbf{D}_o & (\mathbf{D}_e - \mathbf{D}_o)\,\mathbf{J} \\ \mathbf{D}_e - \mathbf{D}_o & -(\mathbf{D}_e - \mathbf{D}_o)\,\mathbf{J} \end{bmatrix} \tag{10}$$

where (i) $\mathbf{D}_e$ is an $M/2 \times M$ matrix with the even-symmetric basis functions; (ii) $\mathbf{D}_o$ is a matrix of the same size with odd-symmetric bases; (iii) $\mathbf{D} = [\mathbf{D}_e^T\mathbf{D}_o^T]^T$ is non-singular; and (iv) $\mathbf{U}$ and $\mathbf{V}$ are $M/2 \times M/2$ biorthogonal matrices. Note that this factorization is general and capable of implementing every critically-decimated FIR filter bank with linear phase filters with the same length and $L = 2M$. In the GenLOT case [12], $\mathbf{D}$ is chosen as the DCT. The inverse transform is given by

$$\mathbf{Q} = \begin{bmatrix} \mathbf{U}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^{-1} \end{bmatrix}^T \begin{bmatrix} \mathbf{D}_e - \mathbf{D}_o & (\mathbf{D}_e - \mathbf{D}_o)\,\mathbf{J} \\ \mathbf{D}_e - \mathbf{D}_o & -(\mathbf{D}_e - \mathbf{D}_o)\,\mathbf{J} \end{bmatrix}. \tag{11}$$

The degrees of freedom are the non-singular matrices $\mathbf{U}$ and $\mathbf{V}$. From (10) and (11) we find

$$\mathbf{Q}^T\,\mathbf{P} = \begin{bmatrix} \mathbf{I} - \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} + \mathbf{A} \end{bmatrix}. \tag{12}$$

where $\mathbf{A} = \mathbf{D}_e^T\,\mathbf{D}_o + \mathbf{D}_o^T\,\mathbf{D}_e$ has order $M \times M$, and represents the time domain alias. Note that time aliasing is independent of $\mathbf{U}$ and $\mathbf{V}$. Furthermore, if integer oversampling ratios are used one can easily check that the PR conditions in (7) are always satisfied regardless of $\mathbf{A}$ in (12). Observe that we may have to adjust the gain of the transforms, according to the oversampling ratio, such that the overall gain is made equal to one. Also, using (7) it is possible to show that PR is not possible with a non integer oversampling ratio using non-trivial choices of $\mathbf{D}_e$ and $\mathbf{D}_o$.

4. COSINE MODULATED TRANSFORMS

In this section we show that a representative class of cosine modulated filter banks does not admit a non integer oversampling ratio. It is in fact based on a biorthogonal extension of the extended lapped transform (ELT) [14], [5] or the ELBT. The conditions for PR with non integer oversampling ratio impose that some coefficients of the prototype window have to be made equal to zero. Certainly, this is a non satisfactory solution since it removes the overlapping of the basis functions, and, hence, the nice properties that come from it.

The basis functions of the ELBT have lengths equal to $L = KM = 2\,K_o\,M$, where K_o is the *overlapping factor*. It can be written as

$$\mathbf{P}_i = \mathbf{\Phi}_i^T \mathbf{H}_i \mathbf{Q}_i = \mathbf{\Phi}_i^T \mathbf{F}_i, \tag{13}$$

for $i = 0, 1, \ldots, K-1$, where $\mathbf{H}_i$ and $\mathbf{F}_i$ are the ith blocks of the prototype windows in the analysis and synthesis, respectively, such that,

$$\mathbf{H}_i = \mathrm{diag}\{h[i\,M],\, h[i\,M+1],\, \ldots,\, h[i\,M+M-1]\}, \tag{14}$$
$$\mathbf{F}_i = \mathrm{diag}\{f[i\,M],\, f[i\,M+1],\, \ldots,\, f[i\,M+M-1]\}. \tag{15}$$

The analysis and synthesis prototype windows are symmetric, that is, $\mathbf{J}\,\mathbf{H}_{K-1-i}\,\mathbf{J} = \mathbf{H}_i$, and $\mathbf{J}\,\mathbf{F}_{K-1-i}\,\mathbf{J} = \mathbf{F}_i$. The other matrix in the product within (13), $\mathbf{\Phi}_i$, is the ith block of the cosine modulation matrix, defined by

$$[\mathbf{\Phi}_i]_{n,k} = \sqrt{\frac{2}{M}}\cos\left[\left(n + i\,M + \frac{M+1}{2}\right)\left(k+\frac{1}{2}\right)\frac{\pi}{M}\right] \tag{16}$$

where $[\mathbf{\Phi}_i]_{n,k}$ means the element at the nth row and kth column of the matrix $\mathbf{\Phi}_i$. The submatrices $\mathbf{\Phi}_i$ have the following properties:

$$\begin{aligned} \mathbf{\Phi}_i\,\mathbf{\Phi}_{i+2\ell}^T &= (-1)^\ell\left[\mathbf{I} + (-1)^{i+1}\mathbf{J}\right], \\ \mathbf{\Phi}_i\,\mathbf{\Phi}_{i+2\ell+1}^T &= \mathbf{0}, \end{aligned} \tag{17}$$

for ℓ an integer.

The product $\mathbf{Q}^T\mathbf{P}$ is used to examine the conditions for perfect reconstruction with N-fold decimators and N-fold expanders. From the equations above we write

$$\mathbf{Q}^T\,\mathbf{P} = \mathbf{D} + \mathbf{A}, \tag{18}$$

where $\mathbf{D}$ is a $K\,M \times K\,M$ diagonal matrix with submatrices

$$\mathbf{D}_{i,j} = \begin{cases} \mathbf{F}_i\,\mathbf{H}_i, & \text{for } i = j \\ \mathbf{0}, & \text{otherwise} \end{cases} \tag{19}$$

for $i = 0, 1, \ldots, K-1$, and $j = 0, 1, \ldots, K-1$. The component $\mathbf{A}$ is a $K\,M \times K\,M$ matrix with submatrices

$$\mathbf{A}_{i,j} = \begin{cases} \mathbf{0}, & \text{for } i-j \text{ odd} \\ (-1)^{i+1}\mathbf{F}_i\mathbf{J}\mathbf{H}_i, & \text{for } i = j \\ (-1)^{\frac{i-j}{2}}\mathbf{F}_i\mathbf{H}_j - (-1)^{\frac{i+j}{2}}\mathbf{F}_i\mathbf{J}\mathbf{H}_j, & \text{otherwise} \end{cases} \tag{20}$$

for $i = 0, 1, \ldots, K-1$, and $j = 0, 1, \ldots, K-1$. After the appropriate overlapping of the blocks, the resulting main diagonal matrix, obtained from $\mathbf{D}$, must be made equal to an identity matrix, as illustrated in the scheme of Fig. 3. The terms outside the main diagonal, related to $\mathbf{A}$, represent time alias and must be made equal to zero.

It is easy to show that the condition for perfect reconstruction with non integer oversampling ratio M/N imposes coefficients of the analysis and synthesis windows equal to zero.

Table 1. Number of constraints for an 8-band ELBT.

N	8	7	6	5	4	3	2	1
$K_o = 1$	8	12	11	11	6	10	5	5
$K_o = 2$	24	43	37	40	18	33	15	14
$K_o = 3$	40	90	67	81	30	60	25	23
$K_o = 4$	56	149	97	126	42	87	35	32

For this purpose, let us start with $(-1)^{K_o}\,\mathbf{F}_{K-2}\,\mathbf{J}\,\mathbf{H}_0$ and $(-1)^{K_o+1}\,\mathbf{F}_{K-1}\,\mathbf{J}\,\mathbf{H}_1$ in submatrices $\mathbf{A}_{K-2,0}$ and $\mathbf{A}_{K-1,1}$, respectively. With N-folders at the input and output blocks of the lapped transform, we implement the overlap shifting those two reverse diagonal matrices according to Equation 7. It is evident that $(-1)^{K_o}\,\mathbf{F}_{K-2}\,\mathbf{J}\,\mathbf{H}_0$ will superpose the component $(-1)^{K_o}\,\mathbf{F}_{K-2}\,\mathbf{J}\,\mathbf{H}_0$, and consequently allow us to mutually cancel the two components, if and only if M/N is an integer. Otherwise, for a non integer oversampling ratio M/N, but perfect reconstruction, we would have to make the two components equal to zero. This would imply that M coefficients in $(-1)^{K_o}\,\mathbf{F}_{K-2}\,\mathbf{J}\,\mathbf{H}_0$ and M distinct coefficients in $(-1)^{K_o+1}\,\mathbf{F}_{K-1}\,\mathbf{J}\,\mathbf{H}_1$ should be made equal to zero. Following the same reasoning for all the remaining blocks with reverse diagonal matrices of the alias component, one would find out that the prototype windows, and, therefore, the length of the transforms, would have to be reduced from an initial length $K\,M$ to M, by zeroing their coefficients, resulting in a short diagonal matrix on $\mathbf{D}$. Therefore, to remove all the time alias we would have to reduce drastically the length of the lapped transform.

Table 1 shows the number of constraints for an 8-band ELBT filter bank. For an integer oversampling ratio M/N, the number of constraints can be determined by $(K-1)(M+N) + M/2 + \lceil N/2 \rceil$. This table also shows an increase in the number of constraints for a non integer oversampling ratio. In fact, one can check that many of these constraints will impose coefficients of the prototype windows equal to zero. It is easy to show that if, on the other hand, the oversampling ratio is an integer, not only time aliasing is always canceled, but there are always enough degrees of freedom to design non trivial prototype windows ($\{h[n]\}$ and $\{f[n]\}$) such that (7) is satisfied. In conclusion, for this representative class of cosine modulated filter banks there is an inverse if and only if M/N is an integer.

5. COMPLEX MODULATED TRANSFORMS

There is an interesting case of modulated filter banks where the entries are allowed to be complex numbers. Such a structure was presented in [15] for example. Basically, it consists of a 2 times redundant ELBT. We show that with this structure the alias component is diagonal and the oversampling ratio can be made integer or non integer.

The basis functions are defined by cosine and sine modulation of the analysis and synthesis windows. The cosine modulation submatrices were introduced already in (16). The sine modulation submatrix, $\hat{\mathbf{\Phi}}_i$, is defined by

$$[\hat{\mathbf{\Phi}}_i]_{n,k} = \sqrt{\frac{2}{M}}\sin\left[\left(n + i\,M + \frac{M+1}{2}\right)\left(k+\frac{1}{2}\right)\frac{\pi}{M}\right]. \tag{21}$$

These submatrices $\hat{\mathbf{\Phi}}_i$ have the following properties:

$$\begin{aligned} \hat{\mathbf{\Phi}}_i\,\hat{\mathbf{\Phi}}_{i+2\ell}^T &= (-1)^\ell\left[\mathbf{I} + (-1)^{i}\mathbf{J}\right], \\ \hat{\mathbf{\Phi}}_i\,\hat{\mathbf{\Phi}}_{i+2\ell+1}^T &= \mathbf{0}, \end{aligned} \tag{22}$$

Table 2. Number of constraints for an 8-band complex ELBT.

N	8	7	6	5	4	3	2	1
$K_o = 1$	4	4	3	3	2	2	1	1
$K_o = 2$	12	11	9	8	6	5	3	2
$K_o = 3$	20	18	15	13	10	8	5	3
$K_o = 4$	28	25	21	18	14	11	7	4

for ℓ integer. Comparing the properties above with the ones for the cosine modulation, in (17), we observe the change of signal in the reverse identity matrix part, that is, $(-1)^{i+1}\,\mathbf{J}$ in (17), and $(-1)^{i}\,\mathbf{J}$ in (22). Now, with two ELBT structures in parallel, one modulated in cosine and the other in sine, when we cascade the direct and inverse transform matrices, without modifying the transform coefficients, we obtain

$$\mathbf{Q}^T\,\mathbf{P} = \mathbf{D} + \mathbf{A} \tag{23}$$

where $\mathbf{D}$ is a $KM \times KM$ diagonal matrix with submatrices

$$\mathbf{D}_{i,j} = \begin{cases} 2\,\mathbf{F}_i\,\mathbf{H}_i, & \text{for } i = j \\ \mathbf{0}, & \text{otherwise} \end{cases} \tag{24}$$

for $i = 0, 1, \ldots, K-1$, and $j = 0, 1, \ldots, K-1$. Note that $\mathbf{D}$ in (24) is not identical to $\mathbf{D}$ in (19). Now, for $M = N$ we already have a 2 times redundant ELBT. The component $\mathbf{A}$ is a $KM \times KM$ matrix with submatrices

$$\mathbf{A}_{i,j} = \begin{cases} \mathbf{0}, & \text{for } i-j \text{ odd or } i = j \\ (-1)^{\frac{i-j}{2}}\,2\,\mathbf{F}_i\,\mathbf{H}_j, & \text{otherwise} \end{cases} \tag{25}$$

This alias component is quite different from the ELBT, for which the alias component has reverse diagonal blocks. Now the reverse diagonal time-domain aliasing terms are canceled when the overlapped blocks are superimposed, and the complex case allows integer or non integer oversampling ratios.

Table 2 shows the number of constraints for an 8-band complex ELBT filter bank. For an oversampling ratio M/N, integer or non integer, the number of constraints can be determined by $(K-1)N + \lceil N/2 \rceil$. Now, in contrast with the result of the previous section, it is possible to design prototype windows satisfying the constraints but without necessarily zeroing the coefficients.

6. CONCLUSIONS

Recent studies of oversampled filter banks have produced some improvements over the well studied critically decimated filter banks, but designs presented in the literature have been limited to integer oversampling ratios. For some popular classes of filter banks we have shown that it is not possible in general to oversample analysis FIR filter banks by non integer factors and expect to find another FIR synthesis bank of the same length to yield PR. The converse is true for integer factors, i.e. one can always oversample the filter bank by an integer factor and expect to find a synthesis bank with similar characteristics. We have investigated the possibility of using non integer oversampling ratios in a structure of two filter banks in parallel. The analysis is carried within a time-aliasing framework developed to analyze non-critically decimated filter banks. We have not been successful in determining the solution for the problem for the general case. However, we have analyzed a few interesting classes of filter banks. As an ongoing project, further research will be undertaken to explore more general solutions.

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